Self-consistent effects of continuous wave output coupling of atoms from a Bose-Einstein condensate

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(February 1, 2008)

We present a self-consistent mean field model of the extraction of atoms from a Bose-Einstein condensate to form a CW atom laser. The model is based upon the Hartree-Fock Bogoliubov equations within the Popov approximation, modified by the inclusion of spatially dependent source and sink terms, which lead to current flow within the condensate. The effects of this current flow are investigated for traps containing Rubidium (repulsive effective interaction) and Lithium (attractive interaction) atoms. The extra kinetic energy associated with this flow is shown to be able to stabilise the condensate in the attractive case against mechanical instability.

PACS Numbers: 03.75.Fi, 05.30.Jp, 67.40.Db

The possibility of producing an intense coherent beam of atoms from a trap containing a sample of Bose-Einstein condensed atoms, a so called atom laser, has attracted much interest in the time since Bose-Einstein condensation was first achieved experimentally in a dilute gas of atoms [1]. To our knowledge little or no work has appeared however, investigating what effects extracting such a beam of atoms would have on the properties of the condensate. In particular, does the extraction of a small proportion of the condensate over some reasonable timescale drastically alter properties such as the condensate profile and noncondensate fraction [2]? Given the great qualitative and quantitative success of mean field theories [3], specifically Hartree-Fock Bogoliubov (HFB) theory [4], in determining the collective excitation spectrum, condensate profile, noncondensate density and noncondensate fraction for trapped Bose gases, we develop a version of the HFB formalism within the Popov approximation [5] (which neglects two-body correlations) [6], in which we include a spatially dependent sink term, which extracts atoms from the condensate. This corresponds to the output coupling of atoms from the trap in some manner. We envisage some form of continuous Raman output coupling [7], but the specific mechanism is unimportant in this treatment. Also included is a stimulated pumping term which corresponds to the Bose enhanced scattering of atoms from the non-condensate into the condensate. The total number of atoms in the trap is kept constant and the noncondensate and condensate profiles are determined self-consistently. This corresponds to an assumption that the trapped atoms are replenished at the same rate as atoms are removed from the condensate via a thermal bath of atoms at a given temperature and that these 'new' thermal atoms equilibrate on a timescale that is small compared to the characteristic timescale of the trap.

This leads to a modified time-dependent Gross-Pitaevskii equation now given by,

$$i\hbar\frac{\partial\Psi(\mathbf{r},t)}{\partial t}=\left\{-\frac{\hbar^2}{2M}\nabla^2+V_{ext}(\mathbf{r})+g[n_c(\mathbf{r})+2\tilde{n}(\mathbf{r})]-\frac{i\hbar}{2}\gamma_c(\mathbf{r})+\frac{i\hbar}{2}\Gamma\tilde{n}(\mathbf{r})\right\}\Psi(\mathbf{r},t). \tag{1}$$

Here we have made the usual decomposition of the Bose field operator, $\hat{\psi}(\mathbf{r},t)$, into condensate and noncondensate parts, i.e., $\hat{\psi}(\mathbf{r},t)=\Psi(\mathbf{r},t)+\tilde{\psi}(\mathbf{r},t)$. The terms involving the interaction strength, $g=4\pi\hbar^2 a/M$, arise from the use of a contact interaction, $g\delta(\mathbf{r})$, where a is the scattering length measured for binary scattering in vacuo. The condensate and noncondensate densities are given by n_c and $\tilde{n}$ respectively. The factor of 2 before the $\tilde{n}(\mathbf{r})$ comes from the equivalence of the direct and exchange energies in the Bose case when one uses a contact interaction. The term involving $\gamma_c(\mathbf{r})$ is the sink term due to the extraction of condensate atoms via the output coupling into the atom laser beam, with a coupling strength determined by γ_c. The Bose enhanced pumping of atoms from the noncondensate into the condensate is given by the term involving Γ, which is a constant determined self-consistently such that the number of atoms scattered from noncondensate to condensate per unit time is equal to the number of atoms extracted from the condensate. Note that this form of the coupling automatically includes the Bose enhancement factor.

We now look for a steady state solution for the self-consistent condensate and non-condensate profiles by looking for solutions of the form $\Psi(\mathbf{r},t)=\Phi(\mathbf{r})e^{-i\frac{\mu}{\hbar}t}e^{i\theta(\mathbf{r})}$. Here μ is the chemical potential and $\theta(\mathbf{r})$ is a position dependent phase. Normally the phase is an arbitrary constant and is set to zero without loss of generality. The phase only becomes a relevant quantity in the context of phase *differences* or gradients. In this problem, with spatially dependent sink and source terms for condensate atoms, one does have a phase gradient which corresponds to a flow of atoms within the condensate.

Making this substitution in the time-dependent Gross-Pitaevskii equation and separating into real and imaginary parts, one obtains two equations. The first is a modified time-independent Gross-Pitaevskii equation;

$$\left\{-\frac{\hbar^2}{2M}\nabla^2 + \frac{\hbar^2}{2M}\left(\frac{\partial\theta(r)}{\partial r}\right)^2 + V_{ext}(r) + g[n_c(r) + 2\tilde{n}(r)]\right\}\Phi(r) = \mu\Phi(r), \tag{2}$$

the second being the differential equation determining the phase gradient

$$\Phi(r)\frac{\partial^2\theta(r)}{\partial r^2} + 2\frac{\partial\Phi(r)}{\partial r}\frac{\partial\theta(r)}{\partial r} + 2\frac{\Phi(r)}{r}\frac{\partial\theta(r)}{\partial r} + \frac{\hbar}{2}\left(\Gamma\tilde{n}(r) - \gamma_c(r)\right) = 0. \tag{3}$$

Here we have made the additional simplifying assumption that the trap is isotropic.

Similar analysis leads to the usual set of coupled HFB equations (within the Popov approximation) given by

$$\hat{\mathcal{L}}u_i(r) - gn_c(r)v_i(r) = E_i u_i(r) \tag{4}$$

$$\hat{\mathcal{L}}v_i(r) - gn_c(r)u_i(r) = -E_i v_i(r), \tag{5}$$

where the Hermitian operator, $\hat{\mathcal{L}}$, is modified by the addition of a kinetic energy term associated with the flow of condensate atoms and is given by

$$\hat{\mathcal{L}} \equiv -\frac{\hbar^2}{2M}\nabla^2 + \frac{\hbar^2}{2M}\left(\frac{\partial\theta(r)}{\partial r}\right)^2 + V_{ext}(r) + 2g(n_c(r) + \tilde{n}(r)) - \mu. \tag{6}$$

The HFB equations define the quasiparticle excitation energies E_i and amplitudes u_i and v_i. The condensate density is defined such that, $n_c \equiv |\Phi(r)|^2$ and noncondensate density such that,

$$\tilde{n}(r) \equiv \langle\tilde{\psi}^\dagger(\mathbf{r})\tilde{\psi}(\mathbf{r})\rangle = \sum_i \left\{|v_i(r)|^2 + \left[|u_i(r)|^2 + |v_i(r)|^2\right] N(E_i)\right\}, \tag{7}$$

where $N(E_i)$ is the usual Bose factor with a temperature determined by that of the external bath of atoms.

These now form a closed set of equations which can be solved self-consistently giving the condensate and noncondensate densities, excitation spectrum and phase gradient.

We now present results for two different regimes. Firstly we consider an isotropic trap with $V_{ext}(\mathbf{r}) \equiv V_{ext}(r) = \frac{1}{2}M\omega_0^2 r^2$, where $M(^{87}Rb) = 1.44 \times 10^{-25}$ kg, and the trap frequency $\nu_0 = \omega_0/2\pi = 200$ Hz. The scattering length for ^{87}Rb is positive (repulsive effective interatomic interaction) and is given by $a \simeq 110a_0 = 5.82 \times 10^{-9}$ m. We consider a trap containing 4000 atoms in the steady state at a temperature of 90 nK. Solving self-consistently for the noncondensate fraction with no output coupling yields 2263 atoms in the condensate and 1737 atoms in the noncondensate at this temperature, i.e. just less than 50 % thermal depletion. We then extract condensate atoms in a localised region at the centre of the trap via a coupling $\gamma_c(r) = Ae^{-\sigma r^2}$. The total number of atoms extracted in a characteristic time $\tau = 1/\omega_0$ is given by,

$$N_{out} = \int d^3r n_c(r)\left[1 - e^{-\gamma_c\tau}\right]. \tag{8}$$

We choose $\sigma = 2$ such that the full-width half-maximum (FWHM) of the coupling is of the order of 1/3 of the FWHM of the condensate. This enables one to induce moderately large phase gradients (and hence current densities, $J = n_c\partial\theta/\partial r$) within the condensate. If one is considering a Raman output coupling scheme, the FWHM would correspond to the focusing of the laser beams. This is of course limited ultimately by the wavelength of the light used. With the parameters used here the focusing would have to be on the order of 1 μm, which is on the extreme limit of achievability. We would like to emphasise that we are looking at a trap containing only 4000 atoms (to which we are limited numerically). Realistic traps are much larger now [8] and hence the focusing in experiments can be much weaker and attain similar results qualitatively to those presented here.

In Fig 1 we present results for the condensate and noncondensate densities and the current density for a range of amplitudes, A, of the coupling strength. Note here that the current density is defined such that current flow towards the centre of the trap is positive. For relatively weak coupling the phase gradient established is small and the density profiles of the condensate and noncondensate are essentially identical to those obtained without any laser output. With increasing coupling strength, the current densities grow monotonically (left panels) with a corresponding increase in the effects upon the densities [9]. The condensate becomes slightly depleted at the centre and the noncondensate density increases slightly. This can be thought of as an effective heating caused by the extraction of the laser beam, but even in this extreme case where we have tried to maximise the phase gradients by taking a small extraction region, the total increase in the number of atoms in the noncondensate as opposed to the condensate is from 1737 atoms with no extraction to 1822 atoms at the maximum extraction rate when $N_{out} \simeq 100$ atoms in time τ.

When the coupling strength is increased further (right panels) the effects upon the densities become more pronounced and the condensate density at the centre of the trap becomes depleted significantly. This actually leads to a reduction in the rate of output of atoms from the trap as the coupling strength is increased. This can also be seen from the fact that the current density for the case with strongest coupling (dashed lines), has actually decreased. There is therefore an optimal strength with which to couple atoms from the condensate so that one can maintain a reasonable yield of atoms without depleting the condensate density excessively. This is equivalent to saying that there is a maximum flow rate for atoms within the condensate.

The second case we wish to consider is the case when the trapped atoms have a negative scattering length, i.e. attractive effective interactions. We therefore consider a trap with $\nu_0 = \omega_0/2\pi = 150$ Hz and $M(^7Li) = 1.16 \times 10^{-26}$ kg. The s-wave scattering length for ^7Li is given by $a = -1.44 \times 10^{-9}$ m. The parameters here correspond closely to those of the Rice group [10].

It is well known that condensates with attractive interactions are only metastable at best and above some critical condensate number become unstable to mechanical collapse, which is characterised by the $l = 0$ collective mode going soft, i.e. the lowest $l = 0$ excitation frequency goes to zero [11]. The critical number for the above parameters has been shown to be $N_c = 1241$ atoms at $T = 0$. It has also been shown that increasing the temperature decreases the stability and decreases the critical number N_c. This decrease manifests itself in a decrease of the $l = 0$ mode frequency with temperature. For example, with 1150 atoms in the condensate and a total number of atoms in the trap determined self-consistently from the HFB equations, the $l = 0$ mode frequency drops from $\omega_{l=0} = 1.523\omega_0$ at $T = 0$ to $\omega_{l=0} = 1.428\omega_0$ at $T = 75$ nK, with 1312 atoms in the noncondensate, viz. a total of 2462 atoms in the trap.

The (meta) stability of the condensate is due to the kinetic energy associated with the trapping potential - there is no stable condensate in the uniform case with attractive interactions. The extraction of atoms via an atom laser sets up a current flow within the condensate which has the effect of increasing the kinetic energy. It is therefore feasible that the extraction of atoms via some method of localised output coupling could stabilise the condensate against mechanical collapse. We therefore apply our model to the case with 1150 atoms in the condensate at 75 nK. In this case $\sigma = 256$ which corresponds to a FWHM of 0.4 μm. This puts one into the uv region for Raman output coupling, which is infeasible. However for this trap a very small output region is required as the characteristic size of the condensate is small. To experimentally investigate these effects one would need to use a trap with a smaller confining potential and hence larger (less dense) condensate. The amplitude of the coupling is 2500 ω_0 which yields a cw laser output of 129.46 atoms$/\tau$. The condensate, noncondensate (magnified by a factor of 20) and current densities are shown in Fig 2. The peak in the current density approximately corresponds, in terms of position to that of that of the point of inflection in $\gamma_c(r)$. The relative energy scales of the kinetic energy due to the current flow and the scale of the interaction energy in this case is of the order

$$\frac{\hbar^2}{2M}\left(\frac{\partial\theta}{\partial r}\right)^2_{max} / g n_c^{max} \simeq 0.2. \tag{9}$$

Even with this relatively small addition to the kinetic energy the frequency of the $l = 0$ collective excitation has been increased form $\omega_{l=0} = 1.428\omega_0$ to $\omega_{l=0} = 1.464\omega_0$. This represents a significant decrease in the softening of the mode and a strong indication of the feasibility of stabilising the condensate against collapse.

In conclusion, we have developed a model based upon well tried self-consistent mean field theory that incorporates the effects of continuous extraction of atoms from the condensate and pumping via scattering of atoms from the noncondensate to condensate. We have investigated the effects of the induced current within the condensate upon the steady state properties of both the condensate and noncondensate. For trapped atoms with repulsive interactions and moderate extraction rates, the effects were not found to be excessively damaging and it appears likely that atoms could be extracted by means of some form of output coupler in a continuous manner, without significantly effecting the steady state properties of the condensate.

In the case of trapped atoms with attractive interactions the spatially localised extraction of an output beam of condensate atoms, setting up a flow of current within the condensate was found to increase the kinetic energy of the condensate. This was found to reduce the softening of the $l = 0$ mode through thermal effects, hence stabilising the condensate against mechanical collapse. It is hoped that for larger traps (weaker confinement) larger phase gradients may be created which, further increasing the kinetic energy, may be able to increase the critical number of condensate atoms for given trap parameters beyond that of the $T = 0$ case with no extraction.

This work was performed at the University of Auckland during a Royal Society Study Visit and we gratefully acknowledge the Royal Society (London), University of Auckland Research Committee, the Marsden Fund of the Royal Society of New Zealand and the UK EPSRC for funding. Thanks go to Tony Wong, Dan Walls and Keith Burnett for many useful discussions and very special thanks in general to the quantum optics group at the University of Auckland for their warm hospitality.

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[9] Note the sharp shoulder in the current density. This is an artifact of a numerical limitation in the calculation. The current density should tend uniformly to zero. The shoulder is caused by a truncation in the calculation of the phase gradient, which tends to infinity as r tends to infinity (but more slowly than the density goes to zero). The only effect of altering the cutoff in these calculations is a small modification in the tail of the noncondensate.

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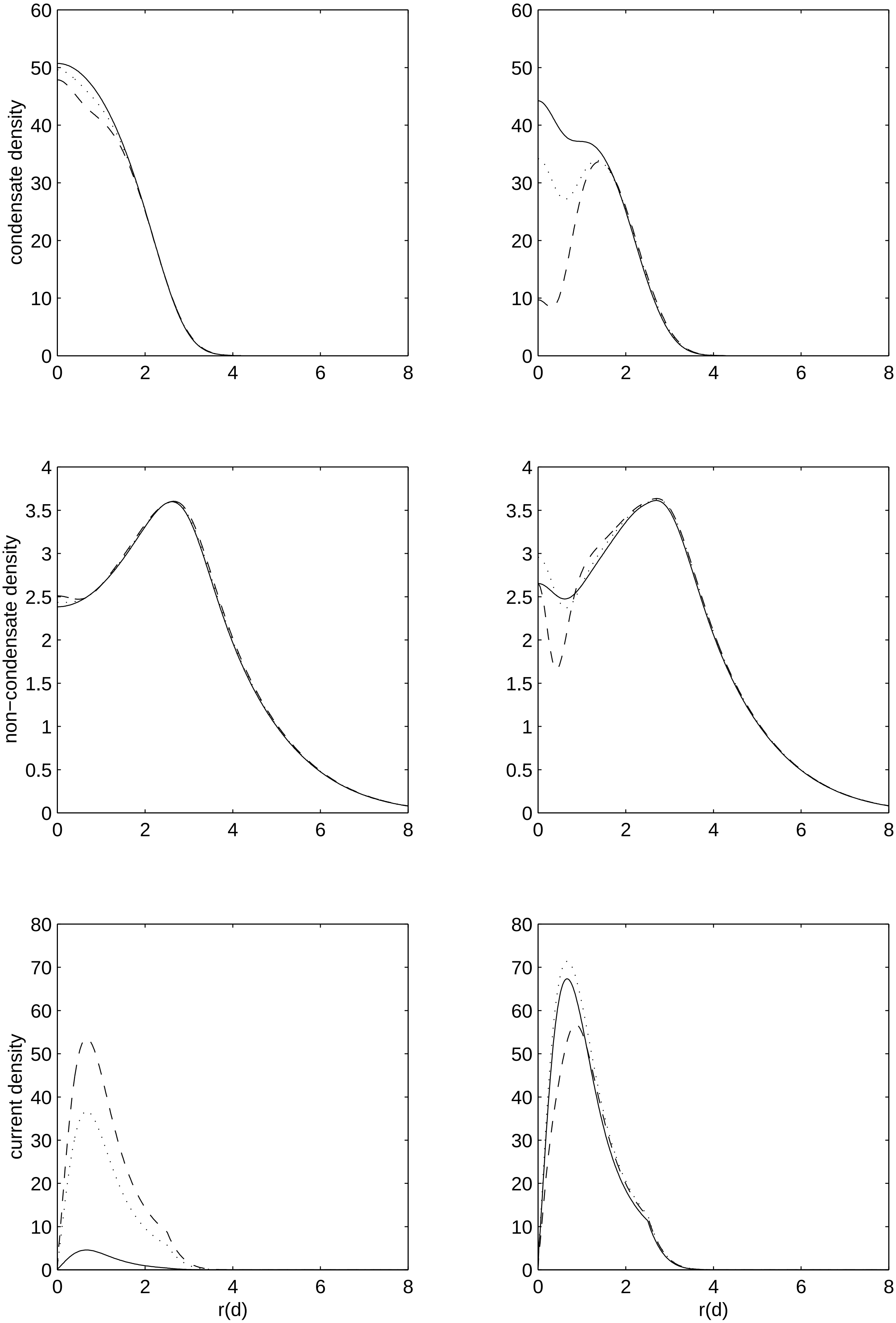
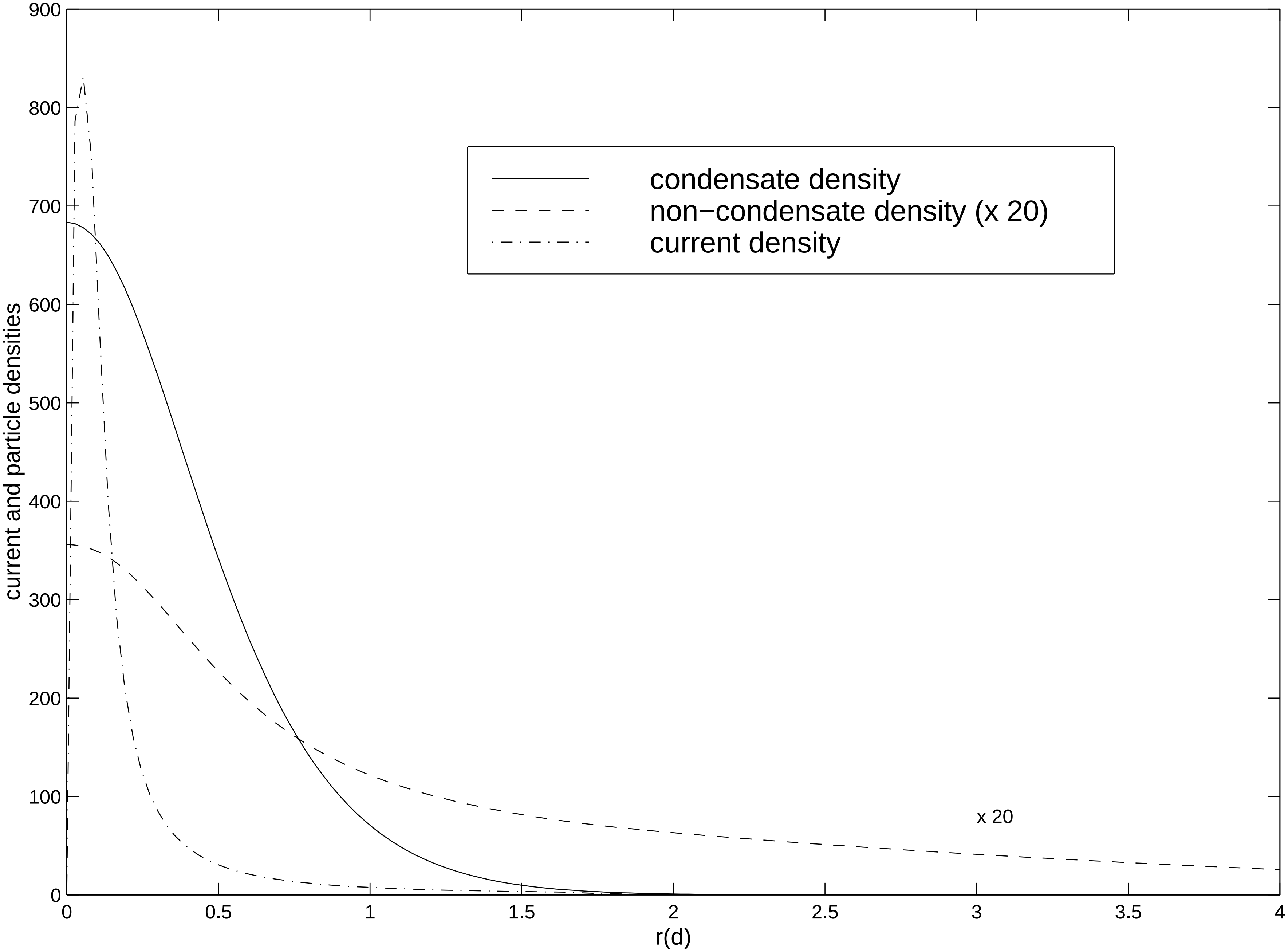
FIG. 1. Condensate and noncondensate densities in units of d^{-3} and current densities ($\omega_0 d^{-2}$) for a trapped gas of 4000 ^{87}Rb atoms at $T = 90$ nK with a spatially dependent output coupling rate given by $\gamma_c = Ae^{-\sigma r^2}$. $\sigma = 2$. Left panels; $A = 10$ (solid line), $A = 80$ (dotted line), $A = 120$ (dashed line). Right panels; $A = 160$ (solid line), $A = 200$ (dotted line), $A = 240$ (dashed line).

FIG. 2. Condensate, noncondensate and current densities for a trapped gas of ^7Li atoms at $T = 75$ nK with 1150 atoms in the condensate and Gaussian output coupling strength $\gamma_c(r)$ centred at the origin.



condensate density
non−condensate density
current density
r(d)
r(d)



condensate density
non−condensate density (x 20)
current density
x 20
current and particle densities
r(d)
0
100
200
300
400
500
600
700
800
900
0
0.5
1
1.5
2
2.5
3
3.5
4